

Self-Focusing Dynamics of Coupled Optical Beams

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We theoretically and experimentally investigate the mutual collapse dynamics of two spatially separated optical beams in a Kerr medium. Depending on the initial power, beam separation, and the relative phase, we observe repulsion or attraction, which in the latter case reveals a sharp transition to a single collapsing beam. This transition to fusion of the beams is accompanied by an increase in the collapse distance, indicating the effect of the nonlinear coupling on the individual collapse dynamics. Our results shed light on the basic nonlinear interaction between self-focused beams and provide a mechanism to control the collapse dynamics of such beams.

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Nonlinear collapse to a singularity and the accompanying richness of physical effects is observed in several fields of physics, such as optics [1], hydrodynamics [2], and plasmas [3]. Self focusing and collapse of optical beams with a power above a critical power P_{cr} [4] in nonlinear Kerr media was first predicted in the 1960's [5]. Over the past decade, with the advent of high peak power ultra-short pulsed lasers, self-focusing dynamics of optical beams have revealed a remarkable richness of spatial and temporal nonlinear phenomena. These include observation of the universal self-similar spatial collapse profile known as the *Townes* profile [6], multiple filamentation [7], self steepening and pulse splitting [8, 9], multiphoton ionization, and supercontinuum generation [9, 10], along with saturation and plasma generation that typically arrest the collapse [11].

Related optical beam interactions within the context of spatial solitons [12, 13] have drawn considerable interest in recent years. Spatial solitons in 1D Kerr media are shown to interact in a particle-like elastic manner, where the number of solitons and the corresponding directions and propagation velocities are conserved [12, 14]. Depending on the relative phase, attractive and repulsive forces and power transfer are observed between interacting solitons [15]. In saturable nonlinear media, which can support (2+1)D solitons, phenomena such as soliton fusion, fission, annihilation, and spiraling occur [16, 17, 18, 19]. The interaction between incoherent solitons, where the medium responds non-instantaneously, exhibits similar effects as those observed with coherent solitons in nonlinear saturable media [20].

In the regime of optical beam collapse in a Kerr medium, where self focusing is dominant and beams do not maintain their spatial distribution, only a few theoretical studies of beam interactions have been reported [21, 22, 23, 24], and to the best of our knowledge no experiments have been performed. While in these initial theoretical studies several qualitative trends, such as repulsion, attraction, and fusion of two beams, were identified, the detailed dynamics and the transition to fusion of two beams, especially when each beam has a power near P_{cr} , has not been explored.

In this Letter, we investigate both theoretically and experimentally the spatial collapse dynamics of two coherently coupled beams in Kerr media. We observe repulsion and attraction between the collapsing beams, and a sharp transition to fusion of the beams, which is dependent on their initial power, spatial separation and relative phase. We further show that this transition, accompanied by a peak in the collapse distance, can be exploited to control and manipulate the collapse dynamics of two coupled beams. Our results shed light on the basic nonlinear interaction between self-focused collapsing beams and are applicable in different scenarios, including that of multiple filamentation and collapse of complex multi-lobe beams such as necklace beams [25].

To investigate the propagation and collapse of two spatially separated beams in Kerr media, we numerically integrate the scalar (2+1)D nonlinear Schrödinger equation (NLSE), neglecting dispersion and high-order terms. These assumptions are reasonable as long as the dispersion length is longer than the nonlinear and diffraction length scales. We express the (2+1)D NLSE in normalized units as

$$i\psi_{\zeta} + \frac{1}{4}\nabla_{\perp}^2\psi + \frac{L_{df}}{L_{nl}}|\psi|^2\psi = 0, \quad (1)$$

where ψ is proportional to the amplitude $A(x, y, z) = A_0\psi(\mu, \nu, \zeta)$ of the envelope of the electric field, $\zeta = z/L_{df}$ is the normalized coordinate in the propagation direction, $\mu = x/r_0$ and $\nu = y/r_0$ are the normalized transverse coordinates, r_0 is the characteristic radius of the input beam, $\nabla_{\perp}^2 = \partial^2/\partial\mu^2 + \partial^2/\partial\nu^2$ is the transverse Laplacian, $L_{df} = kr_0^2/2$ is the diffraction length, $L_{nl} = n_0n_2c|A_0|^2/\lambda$ is the nonlinear length, $k = 2\pi n_0/\lambda$ is the wave number, λ is the vacuum wavelength, n_0 is the linear index of refraction, and n_2 is the nonlinear index coefficient. When the total power $P = (n_0c/2\pi)|A_0|^2 \int |\psi|^2 d\mu d\nu$ in the beam is greater than $P_{cr} = \alpha(\lambda^2/4\pi n_0 n_2)$, where α is a constant that depends on the initial spatial profile [4], the self-focusing term dominates and the beam undergoes collapse.

In the present study, we assume an initial field com-

posed of two spatially separated, parallel, Gaussian beams of the form $\psi(\mu, \nu, 0) = C\{e^{-[(\mu-\Delta_0/2)^2+\nu^2]} + e^{i\varphi}e^{-[(\mu+\Delta_0/2)^2+\nu^2]}\}$, where Δ_0 is the normalized initial spatial separation of the beams, φ is the relative phase between the beams, and C is a normalization constant that depends on the total power. To exclude cases where the two beams are initially unresolvable and hence collapse as a single beam, we assume $\Delta_0 > 1$. Since a collapsing Gaussian beam is known to form a *Townes* distribution carrying one critical power [6], we limit our investigation to the regime of powers $2P_{cr} < P < 10P_{cr}$. This corresponds to powers $P_{cr} < P_G < 5P_{cr}$ for each beam, thus enabling their individual collapse, but restricts the collapse distance to be larger than $0.25L_{df}$.

For a fixed total power of $2.5P_{cr}$, various initial separations Δ_0 , and for a relative phase of either 0 or π (in- and out-of-phase cases), we integrate Eq. (1) with the split-step method and obtain the intensity distribution as a function of ζ . Just before collapse, when the peak intensity reaches 50 times the initial peak intensity, we record the ζ value and the separation between the two collapsing peaks. The distance between the collapse points Δ_{col} and the collapse distance ζ_{col} as functions of the initial separation are shown in Figs. 1(a) and 1(b), respectively. When the separation is large, for both the in- and out-of-phase cases, the collapse dynamics of each beam are essentially independent of the other beam, and the distance between the collapse points is equal to the initial separation. For the in-phase case, as the initial separation is decreased the collapsing beams are attracted towards each other, and their separation decreases with respect to the initial separation. At some critical separation ($\Delta_0 \approx 1.9$) a sharp transition to one collapsing beam is observed. This transition to fusion of the two beams is accompanied by more than a 40% increase in the collapse distance with respect to independent beam collapse. The lengthening of collapse distance represents a critical slowing down, associated with the phase transition between independent collapse and fusion. For the out-of-phase case the behavior is drastically different. When the initial separation is decreased the collapsing beams repel each other and do not undergo fusion to a single collapsing point. This resembles the well-known repulsion observed between two anti-phased spatial solitons [12]. Here the collapse distance is decreased with regard to independent beam collapse.

We further investigate for the in-phase case the dependence of the above transition on the total power. Figure 1(c) shows the distance between collapse points as a function of initial separation for different total powers. As the total power is decreased, approaching $2P_{cr}$ (i.e. one critical power per beam), the critical separation at which the transition occurs increases, and the transition becomes steeper. With a total power of $2.02P_{cr}$, the critical separation is almost three times that for a total power of $10P_{cr}$. As $P \rightarrow P_{cr}$, the distance to collapse is

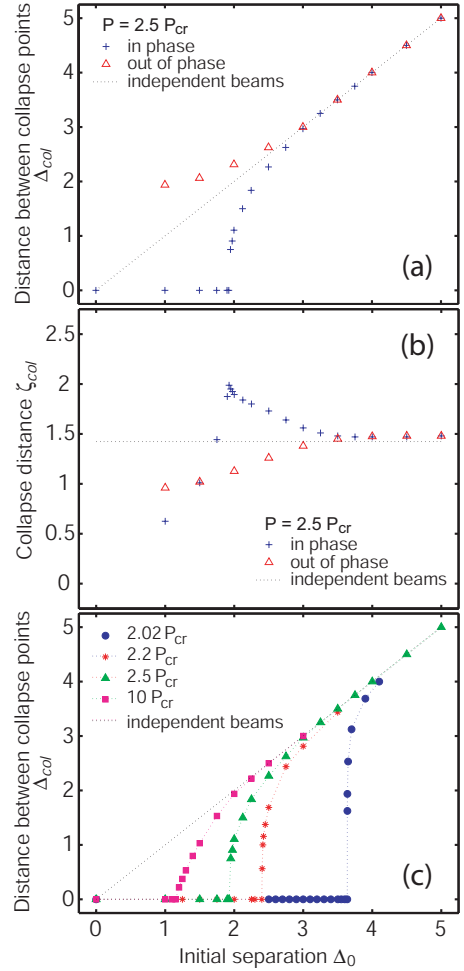


FIG. 1: (Color online) Numerical calculations of the collapse dynamics of two coupled Gaussian beams in Kerr media. (a) Spatial separation between collapse points and (b) collapse distance as a function of initial separation between the beams, for 0 and π relative phase (total power $2.5P_{cr}$); (c) same as the in-phase case in (a) for various input powers.

extended, allowing for longer lengths of mutual attraction and eventually for fusion into one beam (e.g. with a total power of $2.5P_{cr}$ the independent collapse distance is $1.4L_{df}$ while with $2.02P_{cr}$ it is $7L_{df}$, and is more than doubled at the transition region to $18L_{df}$). At relatively large total powers, the transition occurs at smaller initial separations, such that the beams appear to be decoupled for most initial separations. Unlike the case of soliton interactions, these dynamics are governed simultaneously by two competing processes: individual beam collapse and fusion of the beams.

To experimentally investigate the collapse dynamics of this system, we use the arrangement schematically shown in Fig. 2. A pulsed, Ti:Sapphire amplified laser beam, with 1-kHz, 90-fs pulses centered at 800 nm, is passed through a spatial filter to obtain a nearly Gaussian profile. This beam is sent into a two-arm interferometer,

composed of a 50:50 beam splitter and two retro reflectors, such that the two spatially separated beams can be recombined at the same beam splitter. The separation of the two beams is changed by moving one of the retro reflectors in the transverse direction. The temporal alignment needed for coherently combining two short pulses and for controlling their relative phase is achieved by changing the length of one arm with a high-precision motorized stage. The combined beams emerging from the interferometric setup are telescoped down to obtain a 0.29-mm beam waist (ω_0) and then sent into a 28-cm rod of BK7 glass (equivalent to $0.56L_{df}$). The intensity distribution at the input and output faces of the BK7 rod are imaged onto a 12-bit CCD camera. In order to monitor the relative phase of the two combined beams, the far-field intensity fringes of the input field distribution are imaged onto an additional CCD camera.

Initially, one arm is blocked, and the laser power is adjusted so that collapse of a single beam is obtained at the output face of the BK7 rod. The energy per pulse is measured to be $3.6 \mu\text{J}$, equivalent to 40 MW, which is about a factor of 10 higher than expected from the CW numerical calculations for collapse after $0.56L_{df}$ in BK7 glass. We attribute this discrepancy in powers mainly to dispersive effects in the long glass rod and find that this translates into merely a scaling factor for the power and does not affect significantly the spatial dynamics under investigation. This power, at which collapse of a single beam is observed at the output, is held constant throughout the experiments. In practice, due to coherent overlap of the beams on the beam splitter, the total power slightly increased as their separation is decreased. For beam separations greater than $1.5\omega_0$ this increase is calculated to be less than 32%.

After setting the power, we unblock the two arms and for each initial separation of the two beams we record the input and output intensity distributions. This is done for the case in which the beams are temporally misaligned (i.e. the independent beam case) and for both the in- and out-of-phase cases, where the relative phase is zero and π , respectively. Several representative intensity distribu-

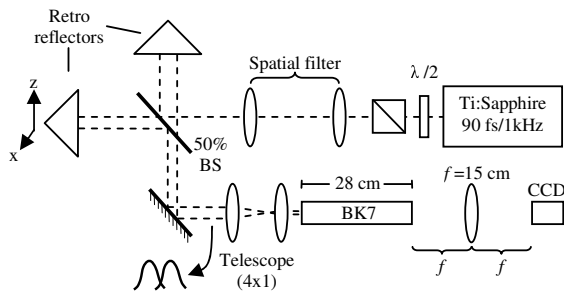


FIG. 2: Ultrafast experimental arrangement for self focusing and collapse of two spatially separated Gaussian beams in BK7 glass.

tions for different initial separations and relative phase are shown in Fig. 3. For the in-phase case, the observed behavior is in agreement with our simulations in which two collapsing points occur at the output when the initial separation is large, whereas only one collapse point is seen when the initial separation is small. This is a clear indication of predicted attraction and fusion between two collapsing in-phase beams. For the out-of-phase case, two collapsing points are always observed at the output, indicating repulsion between the collapsing beams, which is also in agreement with our theoretical results.

From the recorded intensity distributions, we measure the initial separation between the beams at the input face and the corresponding separation between the collapse points at the output face, and the results are plotted in Fig. 4. As expected from the numerical calculations, attraction between the beams and a sharp transition to a single collapse point is observed for the in-phase case. This occurs at a separation distance of $1.75\omega_0$, corresponding to the numerical calculations with $P_{total} \approx 4P_{cr}$ [see Fig. 1(c)], which is in good agreement with the collapse distance. For the out-of-phase case, the beams repel each other, and the separation between the collapse points does not decrease below 0.52 mm, which is equal to $1.8\omega_0$.

In order to experimentally test our ability to dynamically control the collapse of two coupled beams via their relative phase, we slowly sweep the time overlap of the two beams (near the optimal overlap point), thus chang-

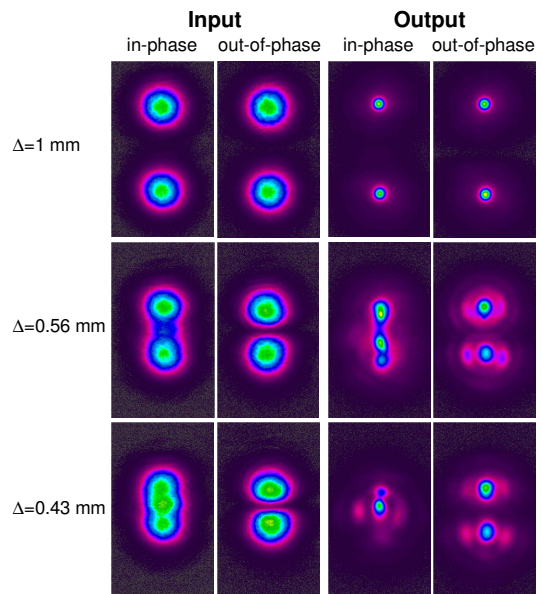


FIG. 3: (Color online) Experimental input and output intensity distributions of two collapsing beams with different initial separations for the in- and out-of-phase cases. The Gaussian beam waist is 0.29 mm, the total energy for the two beams is $7.2 \mu\text{J}$, and the propagation distance in the BK7 glass sample is 28 cm.

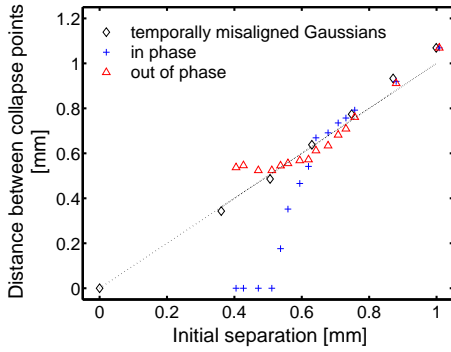


FIG. 4: (Color online) Experimental measurements of the spatial separation between collapse points as a function of initial spatial separation between two beams for 0 and π relative phase and for the temporally misaligned case. Here the initial beam waist is 0.29 mm, the total energy in the two beams is 7.2 μ J, and the collapsing beams are measured after 28 cm of BK7 glass, which corresponds to the distance where independent collapse occurs.

ing their relative phase periodically. This is performed for two different initial separations, the first at a separation of 0.41 mm corresponding to $1.4\omega_0$ where the in-phase case results in fusion into one collapse point, and the second at a separation of 0.54 mm corresponding to $1.86\omega_0$ where the distance between the two collapse points is significantly affected by the relative phase. In the first case the output distribution oscillates between two distinct states with either one or two collapse points. In the second case, the output distribution always has two collapse points, for which the separation increases and decreases periodically. When the relative phase is in the intermediate state (neither 0 or π), the power appears to redistribute between the collapse points in an analogous way to that observed in 1D soliton experiments [13].

In conclusion, we have investigated the collapse dynamics of two spatially separated coupled beams in a Kerr medium. For the in-phase case, we experimentally observe a sharp transition to fusion of the two collapsing beams, which depends on the initial separation and power. We predict that this transition is accompanied by a longer collapse distance with respect to independent collapse. For the out-of-phase case, the beams repel each other and collapse independently. Our results of two-beam interaction serve as a basis for analyzing more complicated scenarios where multiple beams or composite multi-lobe beams undergo self focusing and collapse.

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